MATHEMATICAL MODELING FOR THE CONVECTION BOUNDARY LAYER FLOW IN A VISCOUS FLUID WITH NEWTONIAN HEATING AND CONVECTIVE BOUNDARY CONDITIONS

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ABSTRACT

Problems of convection boundary layer flow are important in engineering and industrial activities. Such flows are applied to manage the thermal effects in many industrial outputs for example in electronic devices, computer power supply and also in engine cooling system such as cooling fins in a car radiator. In modeling the convective boundary layer flow problems, there are four common boundary conditions considered namely as the constant or prescribe wall temperature, constant or prescribe surface heat flux, Newtonian heating and conjugate or convective boundary conditions. Generally, the boundary conditions that are usually applied are the constant/prescribe wall temperature or constant/prescribe surface heat flux. In this study, the boundary condition considered are the Newtonian heating and convective boundary conditions. The Newtonian heating is the heat transfer from the surface is taken to be proportional to the local surface temperature and which is usually termed conjugate convective flow and convective boundary conditions is where heat is supplied through a bounding surface of finite thickness and finite heat capacity. The interface temperature is not known a priori but depends on the intrinsic properties of the system, namely the thermal conductivity of the fluid or solid. The mathematical modeling for the convection boundary layer flow in a viscous fluid is investigated. Three problem have been studied, there are forced convection on a stagnation point flow over a stretching sheet, the extended from the first problem by considering the effects of magnetohydrodynamic in a presence of thermal radiation and the mixed convection on a stagnation point flow past a stretching vertical surface. All of the governing equations which is in the form of non linear partial differential equation from each problem are reduced to ordinary differential equations by using similarity transformation before being solved numerically by using the implicit finite difference scheme known as the Keller-box method. The numerical codes in the form of computer programmes are developed by using the MATLAB software. Six parameter which is the Prandtl number, stretching parameter, conjugate parameter, magnetic parameter, thermal radiation parameter and buoyancy parameter are considered. The features of the flow and heat transfer characteristics for various values of these parameter are analyzed and discussed. It is found that, the increase of Prandtl number, stretching parameter, thermal radiation parameter and buoyancy parameter in an assisting buoyant flow results a decrease in surface temperature. Meanwhile, the trend goes opposite with magnetic parameter, conjugate parameter and buoyancy parameter in an opposite buoyant flow. Futhermore, it is found that the trends for skin friction coefficient, temperature and velocity profiles for convective boundary conditions is quite similar to the Newtonian heating case. On the other hand for heat transfer profiles, it is found that the trends is contrary for all parameters considered except the conjugate parameter.

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a, b, c	Constant
B_0	Uniform magnetic field
C_f	Local skin friction coefficient
C_p	Specific heat
f	Dimensionless stream function
F_x, F_y	Body force in x, y direction, respectively
g	Gravity acceleration
Gr	Grashof number
Gr_c	Critical value of Grashof number
Gr_x	Local Grashof number
h	Heat transfer coefficient
h_s	Heat transfer coefficient for Newtonian heating
h_f	Heat transfer coefficient for convective boundary conditions
k	Termal conductivity
k^*	Mean absorption coefficient
L	Length of plate surface
M	Mgnetic parameter
N_R	Radiation parameter
Nu	Nusselt number
Nu_x	Local Nusselt number
p	Fluid pressure
$\Pr$	Prandtl number
$\Pr_c$	Critical value of Prandtl number

q_r	Radiative heat flux
q_w	Surface heat flux
Re	Reynolds number
Re_c	Critical value of Reynolds number
Re_x	Local Reynolds number
T	Temperature
T_f	Temperature of the hot fluid
T_∞	Ambient temperature
u, v	Velocity component in x, y direction, respectively
u_e	External velocity
u_w	Stretching velocity
U, U_∞	Free stream velocity
x, y	Cartesian coordinate

Greek Symbol

~	Thermal diffusivity coefficient
	Thermal expansión coefficient
¢	Boundary layer thickness
¢$_h$	Velocity boundary layer thickness
¢$_T$	Thermal boundary layer thickness
£	Stretching parameter
£$_c$	Critical value of stretching parameter
¥	Conjúgate parameter
¥$_1$	Conjúgate parameter for Newtonian heating
¥$_2$	Conjúgate parameter for convective boundary conditions

¥$_c$	Critical value of conjúgate parameter
¦	Dimensionless similarity variable
a	Buoyancy parameter or mixed convection parameter
«	Dynamic viscosity
¬	Kinematic viscosity
°	Fluid density
°$_\infty$	Fluid density at ambient temperature
±	Electric conductivity
±*	Stefan-Boltzman constant
²	Shear stress
²$_w$	Surface shear stress
¯	Dimensionless temperature
·	Stream function

Subscript

w	Surface conditions
∞	Outer boundary layer conditions

Superscript

′	Differentiations with respect to ¦

APPENDIX C

LIST OF PUBLICATIONS

A. Journal

Published

1. **Mohamed, M. K. A.,** Salleh, M. Z., Nazar, R. and Ishak, A. 2012. Stagnation Point Flow over a Stretching Sheet with Newtonian Heating. *Sains Malaysiana*. **41**(11): 1467-1473.
2. **Mohamed, M. K. A.,** Salleh, M. Z., Nazar, R. and Ishak, A. 2013. Numerical investigation of stagnation point flow over a stretching sheet with convective boundary conditions. *Boundary Value Problems*. **2013**(1): 1-10.
3. **Mohamed, M. K. A.** and Salleh, M. Z. 2013. Mixed Convection on Stagnation Point Flow over a Stretching Vertical Sheet with Convective Boundary Conditions. Submitted to *Bulletin of the Malaysian Mathematical Sciences Society.*
4. **Mohamed, M. K. A.**, Anwar, M. I., Shafie, S., Salleh, M. Z. and Ishak, A. 2013, Effects of Magnetohydrodynamic on Stagnation Point Flow Past a Stretching Sheet in Presence of Thermal Radiation with Newtonian Heating. Submitted to *Springer Verlag*

B. Proceeding

Presented

1. **Mohamed, M. K. A.**, Nasir, N. M., Khasi'ie, N. S., Jusoh, R., Moslim, N. H., Zaihidee, E. M. and Salleh, M. Z. 2012. Numerical Investigation of Stagnation Point Flow over a Stretching Sheet with Newtonian Heating. *AIP Proceeding of The 2nd International Conference on Fundamental and Applied Sciences 2012 (ICFAS2012)*. Kuala Lumpur Convention Centre (KLCC). Vol. 1482**:** pp 347-350.
2. **Mohamed, M. K. A.,** Salleh, M. Z., Nazar, R. and Ishak, A. (2012), Numerical Investigation of Stagnation Point Flow over a Stretching Sheet with Conjugate Boundary Condition, *The International Conference of Applied Analysis and Algebra (ICAAA 2012),* 20-24 June 2012, Istanbul, Turkey.
3. **Mohamed, M. K. A** and Salleh, M. Z., (2012), Numerical Solution of Stagnation Point Flow over a Stretching Sheet with Newtonian Heating using Keller-box method, *National Conference for Postgraduate Research (NCON-PGR),* 8-9 September 2012, University Malaysia Pahang (UMP)
4. **Mohamed, M. K. A.,** Salleh, M. Z., Nazar, R. and Ishak, A. (2012), Stagnation Point Flow over a Stretching Sheet with Convective Boundary Condition, *20th National Symposium on Mathematical Sciences (SKSM20),* 18-20 December 2012, Putrajaya, Malaysia
5. **Mohamed, M. K. A.**, Anwar, M. I., Shafie, S., Salleh, M. Z. and Ishak, A. (2013), Magnetohydrodynamic effects on stagnation point flow past a

stretching sheet in presence of thermal radiation with convective boundary conditions. *AIP Conference Proceedings of the 20th National Symposium on Mathematical Sciences (SKSM20)*. Putrajaya. **1522**(1): 33-39.

6. **Mohamed, M. K. A.**, Anwar, M. I., Shafie, S., Salleh, M. Z. and Ishak, A. (2013), Effects of Magnetohydrodynamic on Stagnation Point Flow Past a Stretching Sheet in Presence of Thermal Radiation with Newtonian Heating**,** International Conference on Mathematical Sciences and Statistics 2013 (ICMSS2013), 5 - 7 February 2013, Kuala Lumpur, Malaysia.
7. **Mohamed, M. K. A.,** Salleh, M. Z., Nazar, R. and Ishak, A. (2013), Mixed Convection on Stagnation Point Flow over a Stretching Vertical Sheet with Newtonian Heating, The International Conference on Applied Analysis and Mathematical Modeling (ICAAMM 2013), 2-5 June 2013, Istanbul, Turkey

Will be present

1. **Mohamed, M. K. A.,** Salleh, M. Z., Nazar, R. and Ishak, A. (2013), Mathematical Modeling of Mixed Convection on Stagnation Point Flow over a Stretching Vertical Sheet with Convective Boundary Conditions, The 3rd International Conference on Mathematical Sciences (ICMS3), 17-19 Dec 2013, PWTC Kuala Lumpur.
2. **Mohamed, M. K. A.** and Salleh, M. Z. (2013), Effects of Heat Generation/Absorption on a Stagnation Point Flow over a Stretching Surface in Porous Medium with Convective Boundary Conditions, *Malaysian Technical Universities Conference on Engineering and Technology (MUCET)*, 3-4 December 2013, Kuantan, Malaysia.

C. Exposition

Joined

1. **M.K. A. Mohamed,** N. M. Nasir, N. S. Khasi'ie, R. Jusoh, N. H. Moslim, E. M. Zaihidee, M. Z. Salleh (2012), Numerical Investigation of Stagnation Point Flow over a Stretching Sheet with Newtonian Heating, Creation, Innovation, Technology & Research Exposition (CITREx 2012), UMP, 26-28 March 2012, University Malaysia Pahang (UMP)
2. **M.K. A. Mohamed,** N. M. Nasir, N. S. Khasi'ie, R. Jusoh, N. H. Moslim, E. M. Zaihidee, M. Z. Salleh (2013), Magnetohydrodynamic Effects on Stagnation Point Flow Past a Stretching Sheet in Presence of Thermal Radiation with Newtonian Heating, Creation, Innovation, Technology & Research Exposition (CITREx 2013), UMP, 26-28 March 2013, University Malaysia Pahang (UMP)

CHAPTER 1

PRELIMINARIES

1.1 INTRODUCTION

When there is a temperature difference between matters, the thermal transportation (heat transfer) will occur. Basically, the thermal transportation may occur in three conditions which are conduction, radiation and convection. Conduction is the transfer of energy through matter from one particle to the another particle. It is the heat energy transfer and distribution from atom to atom within a medium. Radiation involve the heat transfer from electromagnetic waves come in contact with an object. Meanwhile, the convection is the transfer of heat by the phsysically or actual movement of matter. According to Baehr and Stephen (2006) and Lienhard IV and Lienhard V (2006), heat transfer by conduction and radiation from a solid surface to a fluid named as the convective heat transfer process. In this study, only the convection flow will be considered.

1.1.1 Convective Heat Transfer Process

The word convection refers to heat transfer that occurs between a surface with a moving fluid when both are at a different temperature (Incropera et al., 2006). Generally, convection formed by two mechanisms, namely convection caused by the random motion of molecules (diffusion) and also the energy transferred by the movement of the fluid. The convection heat transfer is a complicated process which it is an actually a surface phenomena. With that, the condition and geometry of surfaces will influence the convection heat transfer process (Darus, 1995).

Convection can be divided into two types which are the forced convection and the free convection (also known as natural convection) (Burmeister, 1983; Pop and Ingham, 2001; Bejan and Kraus, 2003; Kreith et al., 2010). The forced convection occurs when the fluid motion is generated mechanically through the use of a fan, blower, nozzle, jet, etc. Fluid motion relative to a surface can also obtained by moving an object, such as a missile through a fluid. On the other hand, the free convection occurs when the fluid motion is generated by gravitational field. However, the presence of a gravitational field is not sufficient to set a fluid in motion. Fluid density change is also required for free convection to occur. In free convection, density variation is primarily due to temperature changes.

Furthermore, the combination of the free and forced convection is called the mixed convection. The buoyancy parameter $a = \frac{Gr}{Re^n}$ take part as scalar to measure an influence of forced and free convection in a flow with Re as a Reynold's number, Gr as Grashof number and $\left(n > 0\right)$ as a constant. The forced convection is dominant when $a = \frac{Gr}{Re^n} \rightarrow 0,$ while free convection take part as $a = \frac{Gr}{Re^n} \rightarrow \infty$ (Pop and Ingham, 2001; Kreith et al., 2010).

1.1.2 Viscous Fluid

The fluid flow on a flat plate as in Figures 1.1 and 1.2, where u is a fluid velocity in a boundary layer, U_∞ is a stream velocity (free flow outside of boundary layer) and x, y is Cartesian coordinate is considered. The introduction to boundary layer will be discussed later in Section 1.2.

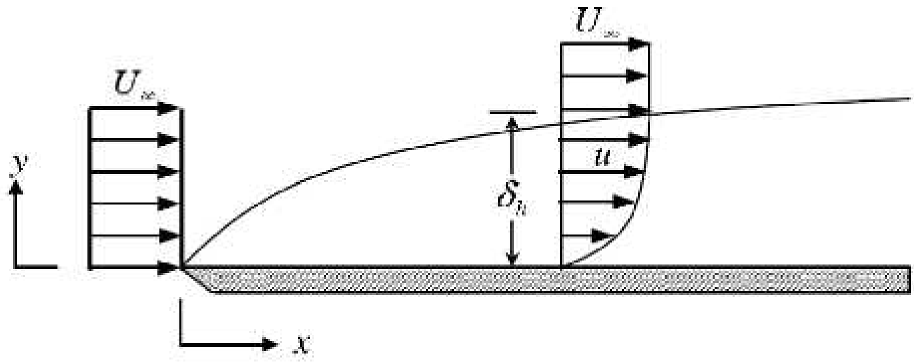

Figure 1.1: The formation of the velocity boundary layer on a flat plate

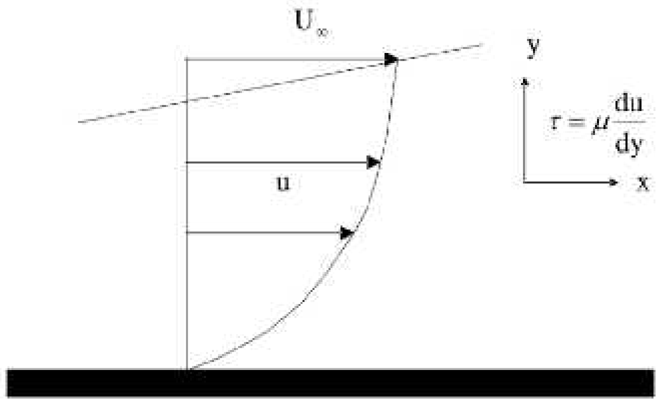

Figure 1.2: Laminar velocity profile on a flat plate.

Fluids can be classifies into Newtonian fluid (viscous fluid) and non-Newtonian fluid. For the Newtonian fluid, the frictional force per unit area, denoted by τ frictional shear stress is linearly proportional to velocity gradient $\frac{du}{dy}$, which

$$\tau = \mu \frac{du}{dy}, \tag{1.1}$$

where μ is a coefficient of dynamic viscosity (Schlichting, 1979; Kreith et al., 2010; Rathore, 2011; Favre-Marinet and Tardu, 2013). The examples of Newtonian fluid are air, water, oil and electrolyte. If the fluids like polimer and paint did not satisfy the Eq. (1.1), then it is classified as non-Newtonian fluid.

Furthermore, fluid flow where the viscosity is neglected is call inviscid flow and « in Eq. (1.1) is assumed to be zero. But, in real day life, the inviscid flow with zero viscosity is not exist.

The word viscous originated from latin word, *viscum* which mean gum. Other word related to it is *viscin* which used to explain about a sticky material. Other than that, viscid (friction) also has been used in order to describe about viscid flow.

1.2 BOUNDARY LAYER THEORY

Boundary layer theory was first introduced by Ludwig Prandtl (1875–1953) on 8th August 1904 in Heidelberg, Germany. The idea is an existence of a thin layer (region) sticks to a surface that are embedded in fluid motion field (Anderson, 2005). This region (thin layer) near surface is called the *boundary layer.*

In introducing the concept of the boundary layer theory, the fluid flow on a flat plate as in Figure 1.3 is considered. Boundary layer is thin layer near the flat plate surface where its viscosity should not be neglected. Also, in boundary layer, the frictional force must be considered while outside the boundary layer, the frictional force is too small and can be neglected (Schlichting, 1979).

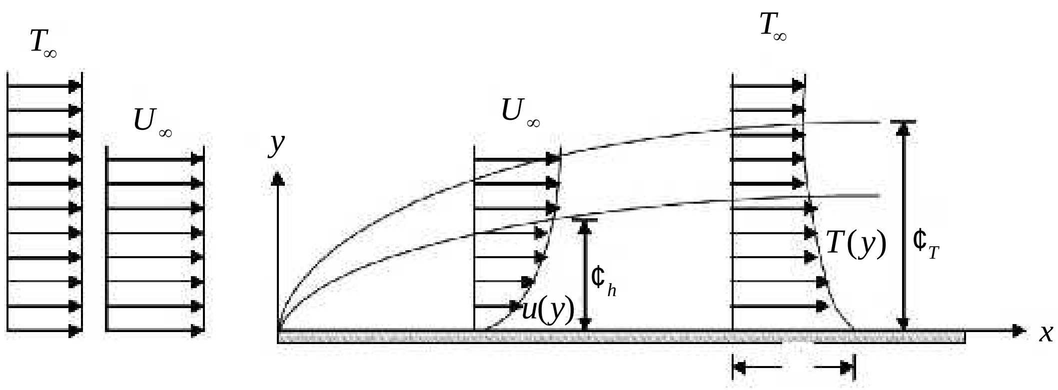

Figure 1.3: Velocity and thermal boundary layer

Boundary layer equations can be derived by setting a few assumptions on the boundary layer flow which are (i) the viscous effects are limited in a boundary layer only. The viscous effects outside of boundary layer are not important; (ii) the boundary layer is smaller than the flat plate surface. If ¢ is a boundary layer thickness and L is flat plate surface, hence ¢ $/L \leq 1$. Also, $x = O(L)$ and $y = O($¢$)$; (iii) the fluid obey the no slip condition on plate surface while free stream velocity at the outside of boundary, which $u(x,0) = 0$, $v(x,0) = 0$, $u(x,\infty) = U_\infty$ and $v(x,\infty) = 0$ where u and v are velocity component in x and y direction, respectively, also U_∞ is free stream velocity; (iv) in the boundary layer, let $u = O(U_\infty)$ (Schlichting, 1979; Ishak, 2008; Ahmad, 2009).

1.2.1 Type of Boundary Layer

The boundary layer can be divided into two which are the velocity boundary layer and thermal boundary layer. By referring to Figure 1.3, it is found that when the fluid molecule attached to the flat surface, we assume that the velocity of the molecule is zero. This zero velocity molecule then delays the movement of other fluid molecule in layer next to it. This process continues until the distance $y =$ ¢$_h$ from the flat surface and after that, this effect can be neglected. By increasing of distance from surface in y, fluid velocity in x component also increases tends the free stream velocity U_∞ outside the boundary layer. This quantity ¢$_h$ is called boundary layer thickness, and usually is defined by y with $u = 0.99U_\infty$. Besides, the velocity profile is changed with respect to the boundary layer (Incropera et al., 2006; Kreith et al., 2010).

Thermal boundary layer is formed when the temperature of the free flow is different with the plate surface temperature. Also, from Figure 1.3, it is found that there is an existence of a region where the temperature change from $T(y)$ at $y = 0$ to T_∞ which is at a free flow outside the boundary layer. The quantity ¢$_T$ is represented the thermal boundary layer thickness. Also, this region can be characterized by the temperature of gradient and heat transfer (Incropera et al., 2006; Kreith et al., 2010).

1.3 DIMENSIONLESS NUMBER

This section will discuss the dimensionless numbers that involve in this study.

1.3.1 Reynolds Number, Re

Reynolds number Re is a dimensionless number which represent the ratio of inertia to viscous forces. Reynolds number can be defined as

$$Re = \frac{U_\infty L}{\neg} = \frac{U_\infty^2 / L}{\neg U_\infty / L^2} = \frac{\text{inertia force}}{\text{viscous force}},$$

where U_∞ is a free stream velocity, is L length of surface plate and $\neg$ as a fluid kinematic viscosity (Rathore, 2011).

Reynolds number used as a criteria for determining the exchanged of the laminar flow to the turbulent flow. If , $Re > Re_c$ where Re_c is a critical value for Reynolds number, then the field flow is considered in a turbulent flow (Darus, 1995). For the flat plate flow, Reynolds number are in between 10^5 to 3×10^6, depending on the surface roughness and the turbulence level in the free stream. The Laminar flow occurs when the Reynolds number is small where the viscous forces are dominant while turbulence flow occurs when the Reynolds value is large where the inertia forces are dominant (Incropera et al., 2006).

1.3.2 Grashof Number, Gr

Grashof number is a dimensionless number which defined as

$$Gr = \frac{g \;\; \Delta T L^3}{\neg^2},$$

where g is gravity acceleration, L as a length of plate surface, ν is the fluid kinematic viscosity, ΔT is difference in temperature in the boundary layer and $= -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)$ is thermal expansion coefficient. In free convection, Grashof number plays the same role as Reynolds number in forced convection. In free convection, the exchange laminar flow to turbulence flow occurs when, $Gr > Gr_c$ where Gr_c is a critical value for Grashof number. For vertical flat plate case, the value of Gr is in between 10^8 to 10^9 (Bejan, 1984; Ozisik, 1985).

1.3.3 **Prandtl Number,** Pr

Prandtl number is dimensionless number which represents the ratio of momentum diffusivity to thermal diffusivity. It is defined as

$$\mathrm{Pr} = \frac{\mu C_p}{k} = \frac{\mu \rho}{k \rho / C_p} = \frac{\nu}{\alpha} = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}},$$

where μ is a dynamic viscosity, C_p as specific heat, k is a thermal conductivity, ρ represent the fluid density, $\nu = \frac{\mu}{\rho}$ as the kinematic viscosity and $\alpha = \frac{k}{\rho C_p}$ is the thermal diffusivity coefficient (Cebeci and Bradshaw, 1988; Darus, 1995).

Physically, a small value of Prandtl number $(\mathrm{Pr} \le 1)$ represent liquid metal which have a high thermal conductivity but low in viscosity, while Prandtl number with higher values $(\mathrm{Pr} \ge 1)$ usually represent oil which have low thermal conductivity but high in viscosity. Specifically, air, electrolyte and water have Prandtl number $\mathrm{Pr} = 0.72, 1, 7$, respectively (Chaudhary and Jain, 2007).

Prandtl number influenced the velocity and thermal boundary layer thickness. For $\mathrm{Pr} \le 1$, it is found that δ_T is bigger than δ_h while for $\mathrm{Pr} \ge 1$, the opposite trend occurs. Furthermore, for $\mathrm{Pr} = 1$, the value of δ_T is seems to be equal to δ_h (Bejan,