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An Analysis of γ Effects to H∞ Filter-based Localization

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Abstract – Instead of using the well-known Kalman filter (H_2 filter), *H*∞ filter which is also known as minimax filter, can be used to estimate robot location as part of simultaneous localization and mapping problem. As γ can affects the *H*∞ filter performance, the variation of its selection are demonstrated to grasp the overview of the estimation performance. The analysis is divided into two categories; non-moving mobile robot and moving robot for different values of γ. The comparison of the performance between *H*∞ filter and Kalman filter is included to differentiate the capabilities of both filters in the localization problem. Simulation results show that, *H*∞ filter exhibits better outcomes than the Kalman filter and thus provides another available methods for the solution of SLAM problem.

Keywords – gamma, *H*∞ filter, Kalman filter, localization.

1. Introduction

Being designed in 1960's, Kalman Filter is known among the best minimum variance linear unbiased estimator using the Minimum Mean Square Estimation (MMSE) approach and a recursive filter for a system [1, 2]. Succesfully applied for nowadays numerous applications, it is a statistical filter that requires modeling the process and measurement errors to have Gaussian white noise with zero mean. Since introduced, Kalman filter been widely used for numerous application worldwide such as in robotics, hydrological and environmental systems, navigation system etc. Unfortunately, it works well under some certain conditions. Above all of its advantages of resulting the smallest standard deviation of estimation error, Kalman filter suffers from some deficiencies such as all noise must be zero mean, and the requirement a priori knowledge for noise standard deviation for both process and measurement noise.

Despite of Kalman filter reputation among decades, some applications still experience problem and need further attention for development. Speaking in the field of autonomous system, estimation problem is a nontrivial issue to be apprehend. Even though Kalman filter may still provide good estimation for some systems, better estimator is still being demand to increase certain autonomous system performance. Practically noise is in uncertain manner and need to be considered carefully when designing a system. Thereby, rather than depending on assumption of Gaussian white noise with zero mean, it is wise decision to model a system that able to take into account worst case of noise or when the statistics of noise is violated. Hence, the development of *H*∞ filter.

In the research of simultaneous localization and mapping (SLAM) [3-9], the process error and measurement errors are often to be non-Gaussian errors and might be in unknown statistics. Therefore, it is a challenge to determine and obtain correct information of robot localization problems and landmarks estimation to build up and achieve a world representation of unknown map. To overcome such shortcomings for a systems of unknown noise characteristics and uncertainties, *H*∞ [8 , 10] is being proposed as a role of an estimators that able to tolerate with a robust system.

2. *H*∞ Filtering: A Robust Filter

Robustness is one of the important criteria to be fulfilled in design a systems that needs a good control and estimation. Firstly introduced around 1987's by Mike Grimble, [2] the only uncertainties remains is in form of bounded energy noise signal in which there is no uncertainty in the matrices of system state space model. In the *H*∞ filtering, the noise inputs are deterministics signals and required to be energy- bounded with no other knowledge of noise is needed. It's guarantees that the energy gain from the noise inputs to the estimation error is less than a certain level.

H∞ filtering has been formulated into 3 main approach such as the Game approach, Riccati Approach and Interpolation theory approach. In contrast with the Kalman Filter, these kind of approach leads to different kinds of equations which lessen its popularity rather than the Kalman filter which is more easier and simpler to be applied. *H*∞ filter is also need some tuning to obtain a good system performance which will be clearer through this paper. Eventually, it is worth to tune the filter to attain better performance to a level of desired outcome. The Riccati approach is being applied in this paper. One of the latest researches conducted on the *H*∞ Filter was conducted by L. Cheng et al. [11] that investigate the wireless sensor network for indoor mobile robot localization. They proposed the mixed Kalman and *H*∞ filter method in delivering the best estimation results especially for distance measurement which can overcome the Non Line of Sight (NLOS) problem. It is also shown in their results that the hybrid system has surpassed the normal EKF, *H*∞ filter.

2.1 Prediction, Filtering and Smoothing

Similar to Kalman filtering, there are some other important keywords to be remind concerning about the estimation problem using the $H\infty$ filtering problem.

- *Filtering*: Estimation of signal process $x(t)$ at time n based on measurement $z(1), z(2), \cdots, z(n)$.
- *Prediction*: Estimation of signal process $x(t)$ at future time point, that is beyond time frame of measurement $z(1), z(2), \cdots, z(n)$.
- *Smoothing*: Estimation of signal process $x(t)$ at time before n and use subsequent measurement up to time n to "smooth" the estimation error.

3. Foundation of $H\infty$ Filtering Problem

The $H\infty$ filtering problem or the minimax estimation problem is represented in this section. Begin considering a linear systems described by the state space form,

$$\dot{x}(t) = Ax(t) + Bw(t), \quad x(0) = x_0 \tag{1}$$

$$y(t) = Hx(t) + Dv(t) \tag{2}$$

$$z = Lx(t) \tag{3}$$

where $x(t) \in \Re^n$ is the state, x_0 an unknown initial state, $w_t \in \Re^l$ the noise signal, $z_t \in \Re^p$ the measurement, and $s_t \in \Re^r$ is a linear combination of state variables to be estimate under horizon [0, T], where $T > 0$ using the measurements $z(\tau), 0 \le \tau \le t$. If L matrix is an identity matrix, x will be the estimation to be obtained.

Assumption 1: $R \triangleq DD^T > 0$

Do note that above assumption is similar to the standard Kalman filter assumption filtering where all components of the measurement vector are assumed to be corrupted by noise.

Assumption 2: Bounded noise energy;

$$\sum_{t=0}^{N} \|w_t\|^2 < \infty, \quad \sum_{t=0}^{N} \|v_t\|^2 < \infty$$

From $\Sigma_0 > 0, Q_t > 0$, and $R_t > 0$ which is the weighting matrix for x_0, noise w_t, and v_t respectively, for $\gamma > 0$, a finite horizon $H\infty$ filtering problem is a problem that must be satisfying the estimation of $\hat{z}_t = \hat{z}_t^*$, $t = 0,1,2,\cdots,N$. This is shown as

$$\sup_{x_0, v, w} = \frac{\sum_{t=0}^{N} \|z_t - \hat{z}_t\|^2}{\|x_0 - \overline{x}_0\|^2_{\Sigma_0^{-1}} + \sum_{t=0}^{N} \|v_t\|^2_{R_t^{-1}} + \sum_{t=0}^{N} \|w_t\|^2_{Q_t^{-1}}} < \gamma^2 \tag{4}$$

The above equation represent that the ratio of estimation error noise and the noise energy bounded for all noise is less than a specified values which is γ in this problem. It is being proved that if the bigger the value of γ, the filter will be reacting familiar to Kalman filter.

A cost function can being applied to look from the aspect of Finite Horizon a priori and a posteriori H∞ filtering problem as shown below.

$$J(\hat{z}; x_0, v, w) = \sum_{t=0}^{N} \|z_t - \hat{z}_t\|^2 - \gamma^2 \left(\|x_0 - \overline{x}_0\|^2_{\Sigma_0^{-1}} + \sum_{t=0}^{N} \|v_t\|^2_{R_t^{-1}} + \sum_{t=0}^{N} \|w_t\|^2_{Q_t^{-1}} \right) \tag{5}$$

where $\hat{z} = (\hat{z}_0, \hat{z}_1, \cdots, \hat{z}_N)$, $v = (v_0, v_1, \cdots, v_N)$, and $w = (w_0, w_1, \cdots, w_N)$. In comparison to (1), it is explicitly clear that,

$$\max_{x_0, v, w} J(\hat{z}; x_0, v, w) < 0, \qquad (x_0 - \overline{x}_0, v, w) \ne 0$$

or

$$\min_{\hat{z}} \max_{x_0, v, w} J(\hat{z}; x_0, v, w) < 0, \qquad (x_0 - \overline{x}_0, v, w) \ne 0$$

which is a minimax problem consisting of two main player, $\hat{z}$ and (x_0, v, w). $\hat{z}$ will be minimizing $J(\hat{z}; x_0, v, w)$ while (x_0, v, w) will be maximizing $J(\hat{z}; x_0, v, w)$.

Rearranging above statements, it can be concluded that the $H\infty$ filtering problem can be stated as follows;

Given a prescribed level of "noise" attenuation $\gamma > 0$ and an initial state weighting matrix $\Sigma_0 = \Sigma_0^T > 0$, find linear causal filter F such that in the finite horizon case, $J(\hat{z}; x_0, v, w) < \gamma$ when $x_0 = 0$.

The resulting filter will be known as $H\infty$ suboptimal filters which is said to achieve a level of noise attenuation, γ.

In this paper, the a posteriori output of filter is being observed for comparison purposes with Kalman filter later on.

Assumption 3: Rank $F_t = n$, for all $t = 0,1,\cdots,N$.

If the above Assumption 3 is fulfilled, for a solution of a posteriori $H\infty$ filtering problem to be exist, the Riccati equation

$$P_{t+1} = F_t P_t \psi_t F_t + G_t Q_t G_t^T, \qquad P_0 = \sigma_0 \tag{6}$$

$$\psi_t = I_n + \left(H_t^T R_t^{-1} H_t - \gamma^{-2} L_t^T L_t \right) P_t \tag{7}$$

holding a positive definite solution and

$$\hat{P}_t^{-1} - \gamma^{-2} L_t^T L_t > 0, \qquad t = 0,1,\cdots,N \tag{8}$$

At this time, for $\gamma > 0$ suboptimal $H\infty$ filter are given by following equation:

$$\hat{z}_t^* = L_t \hat{x}_{t|t}, \qquad \hat{x}_{t+1|t} = F_t \hat{x}_{t|t} \tag{9}$$

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t [y_t - H_t \hat{x}_{t|t-1}], \qquad \hat{x}_{0|t-1} = \overline{x}_0 \tag{10}$$

$$K_t = P_t H_t (H_t P_t H_t^T + R_t)^{-1} \tag{11}$$

4. Simulation Result and Discussion

In order to understand and evaluate the performance of $H\infty$ filter, some simulation is being carried out to observe the filter characteristics with some comparison with Kalman filter. Following simulation setup is being defined to obtain and achieve a good comparison:

- Noise are Gaussian white noise
- Zero mean noise
- Simulation are in planar world
- Landmarks are stationary
- Process noise, $Q = 0.03^2 * I(6)$
- Measurement noise, $R = 0.3 * I(2)$

4.1 Case 1: $H\infty$ Filter Performance Analysis

Here are some of the results obtained when the value of γ is changed. From the theoretical aspects, the bigger the value of γ, the results will be close to the Kalman filter characteristics as shown on Fig. 1. As shown on this figure, it is very critical in choosing the γ value. The significant value of γ can handsomely provides good estimation results, hence better accuracy (note that the true value is 1 for Fig.1, Fig.2, Fig.3, Fig.4 and Fig.5).

As been shown by the figure, the estimation varies as the γ changed but remains stable at a constant value after it reach some value. If the characteristic doesn't change hereafter, this feature is said to be close to the Kalman filter characteristics. Although that it is seems to be very small change of estimation value between γ, the characteristics will be slightly change when employing bigger noise effect to the systems. As can be seen, the γ = 10 shows some unexpected outcome for bigger noise effect than γ = 100, 1000 and still under analysis to determine its cause. This is shown later on Fig. 8 and Fig. 9.

Furthermore, by analyzing these characteristic in changing the values of γ, it is found that, the results will be diversely changing after below the value of 0.5. From Fig. 2, when the smaller value of γ applied on the filter, the estimation become more unstable and changing frequently due to more noise effect included into the system. The smaller the γ, $H\infty$ filter will become more sensitive to the measurement noise. Consequently, the solution for sub-optimal $H\infty$ filter will be not exist and systems cannot function well. This value will be an important reference point for designing an appropriate $H\infty$ filter to achieve certain desired output.

On the other hand, interestingly comparing these γ to acquire best γ value, γ = 1.01 gives the best results as shown on the Fig. 3. However, do note that the value may abruptly change for other conditions that are not specified here.

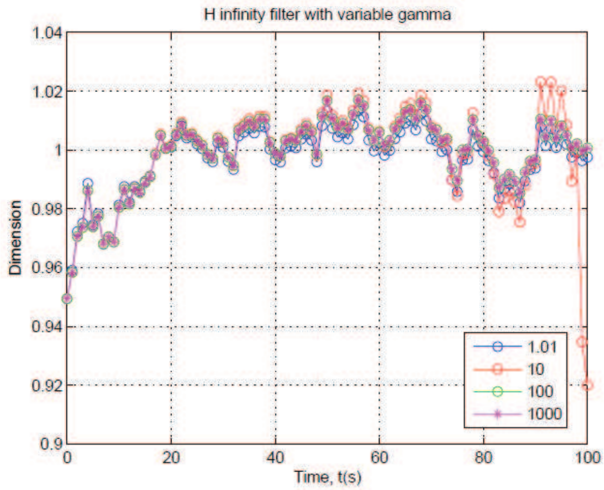

Fig. 1. $H\infty$ characteristics.

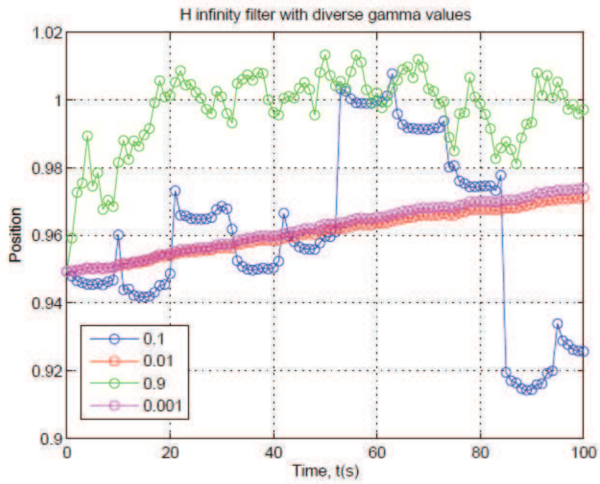

Fig. 2. $H\infty$ γ characteristics: Diverse γ values.

The investigation goes through the evaluation for noise effect to the system. Noise applied to the system is change in a manner to determine and examine its effects to $H\infty$ filter behavior. Results are shown below. As been shown on Fig. 4, the bigger value of noise will rapidly change the behavior of the filter or estimation. In contrast, the smaller the noise, estimation value will be approximating the true value of the systems.

4.2 Case 2: Non Moving Mobile Robot Observing One Landmark at World Coordinate System

The comparison to the well-known Kalman filter is presented. A non-moving case of a mobile robot that simultaneously observing one landmark and at the same time localizing itself to the unknown environment is being studied using an extended kalman filter and the robust $H\infty$ filter.

As shown on Fig. 5, Kalman filter converge faster than $H\infty$ filter. From the characteristics shown on this figure, $H\infty$ filter estimation error is more reliable due to it approximately near to the true value than the Kalman

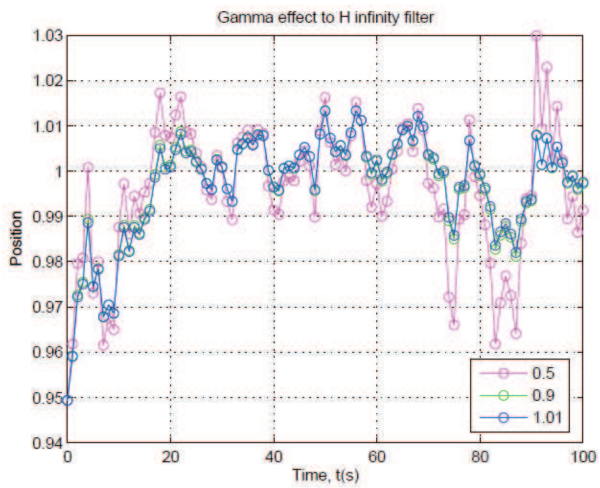

Fig. 3. $H\infty$ γ characteristics: Best γ values.

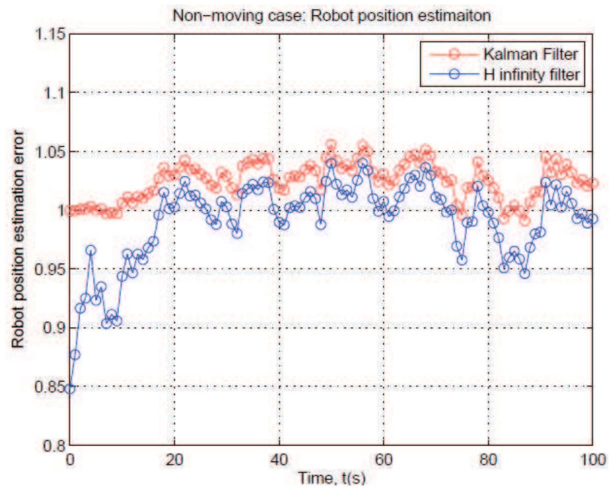

Fig. 5. $H\infty$ filter non-moving case: Robot position estimation.

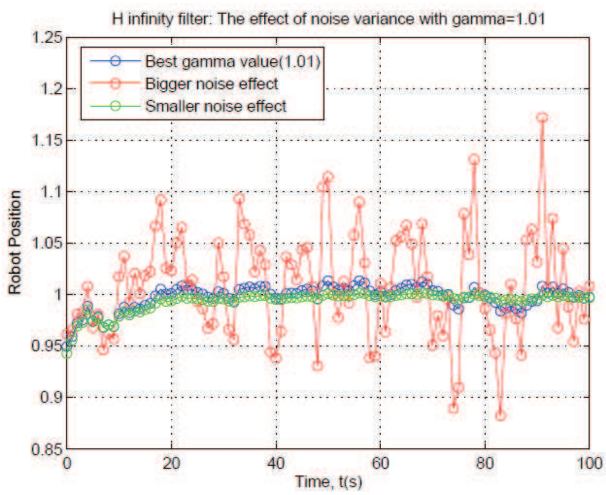

Fig. 4. $H\infty$ filter: Noise effect to the system.

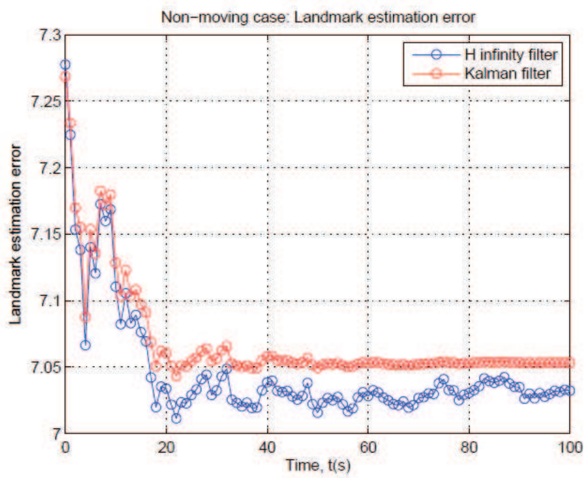

Fig. 6. $H\infty$ filter non-moving case: Landmark estimation with 7 as true value.

filter estimation for true position estimation. Analysis from the average value of both filters resulting below consequences;

- Mean calculation for Kalman filter of robot location = 1.0669
- Mean calculation for $H\infty$ filter of robot location = 0.9833

Furthermore, the $H\infty$ filter oscillates around its true value and better than Kalman filter that is somehow still trying to approximate the true value.

As in Fig. 6, the estimation using $H\infty$ filter surpassed the estimation using Kalman filter for landmark estimation. The characteristics express that the use of $H\infty$ filter gives a promising results than Kalman filter especially in real and practical situations although that it oscillates bigger than the Kalman filter.

Analyzing the uncertainty, it is found that, the landmark uncertainty is decreasing as many more observations are made to the system. Do note that, in this paper, the relative location between robots and landmarks is the objective. At the beginning, $H\infty$ filter shows small deviation of landmark estimation uncertainty rather than Kalman filter. As time passed, even though it is seems that Kalman filter shows a constant output at the end of the simulation, the real landmark estimation value is still not the true value, as stated above in terms of it average estimation.

Based on these two results of above case, this gives an induction and probability of better performance and estimation for a robot observing more and more landmarks while also encouraging outcome for a case where moving mobile robot taking relative observation from some landmarks.

The result for t=10000s simulation time for γ = 0.01 and γ = 0.001. The estimation error fluctuates at approximately 1.05 which is a big error from its true value, i.e. 1.

As for looking into the correction of error for both filters, below figure shows the difference between these filters. From this figure, it is clear that Kalman filter tries to minimize the variance of estimation error while the $H\infty$ provide an uniformly small estimation error for the systems (the big variance between data in each step shows that the estimation error is being suppress under certain level).

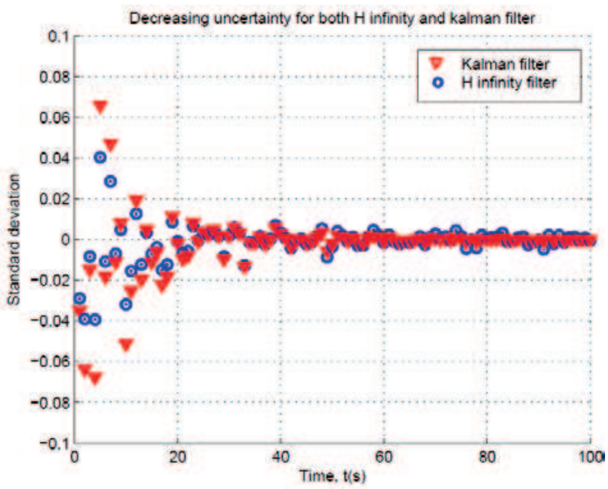

Fig. 7. Uncertainty decreasing: Landmark estimation.

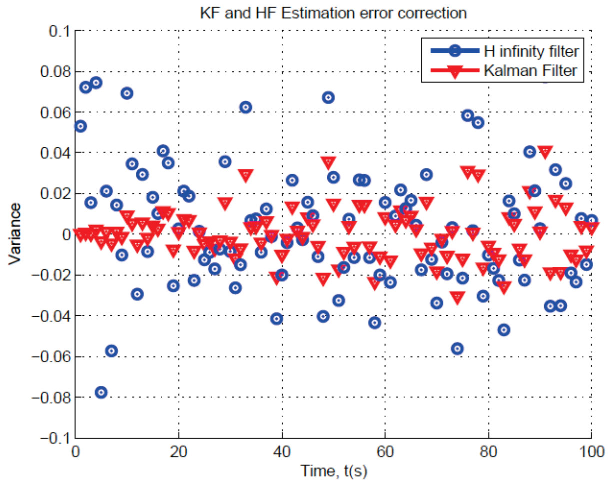

Fig. 9. Kalman filter and *H*∞ filter error comparison.

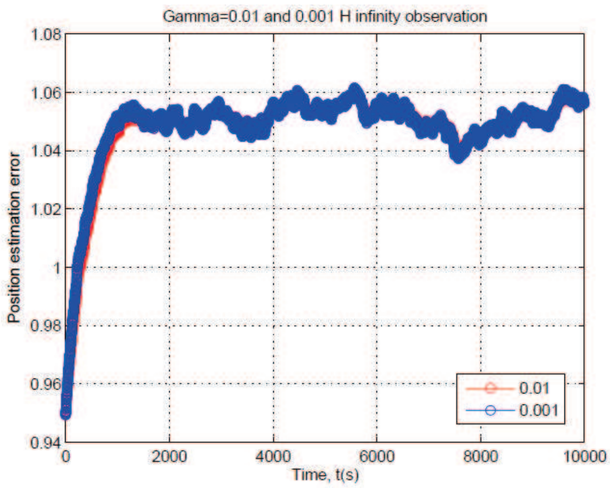

Fig. 8. γ characteristics of *H*∞ filter for γ = 0.01 and γ = 0.001.

5. Conclusion

Above mentioned results determined the promising results using the *H*∞ filter as an alternative way for estimation problem. This approach is still new and may need further improvement and development to achieve even more stable and encouraging results than being attained in this paper. Even though the development of *H*∞ filter is widely diversely approach among researchers, it will be an interesting researches by researchers to extent its advantages for an enormous application widely over the world such as target tracking, system identification, localization, etc. Despite some of its inadequacy, it provides a good estimation rather than other Kalman filter algorithm, EM algorithm and other existed algorithm.

From this paper, it is clear that *H*∞ filter are capable to predict and approximate linear and non-linear system that has wide coverage and variety of noise and proven to be useful for Simultaneous Localization and Mapping Problem (SLAM) that recently being explore and study. However, one should note that, *H*∞ filter are more sensitive at lower level of γ to design a proper system.

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