FATIGUE LIFE ESTIMATION BASED ON CONTINUUM MECHANICS THEORY WITH APPLICATION OF GENETIC ALGORITHM

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ABSTRACT

In recent years research on fatigue estimation methods has focused on developing the capability to handle complex multiaxial loading conditions. The present study presents the development of a new fatigue life estimation model using the concepts of continuum mechanics with a critical plane based approach, where a genetic algorithm is used to estimate the coefficients of the model. Experimental fatigue lives for steel alloy C40 for in-phase and out-of-phase loading conditions are used to calibrate and analyze the accuracy of the proposed model. Finite element analysis is used to determine the experimental fatigue life of steel alloy C40 published in literature for validation purposes. The proposed model is simple to apply as it does not need to determine any new material constants or properties for the new model. Fatigue life prediction from the proposed model shows good agreement with experimental results for in-phase and out-of-phase multiaxial loading.

INTRODUCTION

Since 1860 and Wohler's fatigue investigations, mechanical design has had a major influence on fatigue life determination experiments and predictions [1, 2]. Huge efforts have been made by researchers to safely assess the fatigue life of mechanical components under time-variable loadings [3]. Correctly estimating fatigue life in real components is a complex process having a large number of variables to deal with to avoid unwanted and critical failures [4, 5]. The reliability of a fatigue estimation technique depends on its ability to model damage due to non-zero superimposed static stresses, the degree of multiaxiality in the stress field and effects of stress concentration phenomena [6-9]. Fatigue life is difficult to estimate especially in the cases of cyclic or random multiaxial loading conditions, as damage depends on all of the stress components and their variations during the whole time period of load application [6, 10]. To make the results close to reality, the calibration of such an engineering fatigue assessment method should be based on pieces of experimental information that can be easily obtained through tests run in accordance with the pertinent standard codes [4, 6, 11-13]. The stress analysis is conducted to correctly estimate fatigue damage by directly post-processing simple linear elastic finite element (FE) models [14].

Multiaxial loads, which can be in-phase (proportional) or out-of-phase (non-proportional), are common for many components and structures [15]. Even under uniaxial loads, multiaxial stresses often exist, although typically in-phase, due to

geometric constraints at notches. Such multiaxial loads and stress states are frequently encountered in many industries, including automotive, aerospace, and power generation, among others [15-17]. Many scientific centers and researchers are dealing with this issue, making new tests and developing models for description of the fatigue phenomena. However, still no universal solution has been found [18]. In this paper a new fatigue life estimation model has been proposed with the capability of handling multiaxial loading conditions.

METHODS AND MATERIALS

The concept is based on a continuum approach in which theory is formulated by means of evolution equations, i.e., incremental relations and not changes per cycle. The continuum theory contains in itself damage accumulation during arbitrary load histories, proportional or not, and it thereby avoids cycle counting techniques [19]. As mentioned in the literature, if the fatigue life assessment is to be performed under loading conditions close to real loading conditions, long-term histories of a random character are used. Then the application of cycle counting methods leads to an over-complicated lengthy solution, which cannot be applied in practical engineering analysis [20]. A similar approach has been adopted by Brighenti and Carpinteri [4], while developing their endurance function based model, and a dimensionally balanced equation was proposed based on stress invariants. However, in the authors' opinion, because of the use of invariants in their model it ignores information on the direction of crack growth, i.e., the location of the critical plane, which limits the model to be used only in the crack initiation portion of total fatigue life.

To model the evolution equations, previously published models have been studied to identify the parameters required to capture the state of material under load in cases of various material and loading conditions. Strain-based models are considered for this study as they are more robust in capturing high-cycle fatigue and also low-cycle fatigue where plasticity may occur, and the models under study have either gained some degree of acceptance or are representative of a larger group of related models [21]. A model has been proposed based on cyclic shear and normal strain on the plane of maximum shear to define the stress parameter [22, 23] as in Eq. (1). Cyclic shear strains will help to nucleate cracks, and normal strain will assist in their growth.

$$\frac{\Delta\gamma_{max}}{2} + S\Delta\varepsilon_n = (1.3 + 0.7S)\frac{\sigma_f^{'} - 2\sigma_{n,mean}}{E}(2N_f)^b + (1.5 + 0.5S)\varepsilon_f^{'}(2N_f)^c \qquad (1)$$

where $\Delta\gamma_{max}$ is the maximum shear strain range, $\Delta\varepsilon_n$ is the normal strain range on the plane experiencing $\Delta\gamma_{max}$, S is a material dependent parameter representing the influence of normal strain on material crack growth, N_f is fatigue life, σ_f', ε_f', E, b, c are material properties having their usual meanings. Building on the work on Brown and Miller [22], Fatemi and Socie [24] proposed to replace the normal strain term by the normal stress, arguing that normal stress can be used to describe mean stress and non-proportional hardening effects (Eq. (2)). Critical plane models that include only strain terms cannot reflect the effect of mean stress or strain path dependent hardening.

$$\frac{\Delta\gamma}{2}\left(1 + k\frac{\sigma_{n,\max}}{\sigma_y}\right) = \frac{\tau_f^{'}}{G}\left(2N_f\right)^{b\gamma} + \gamma_f^{'}\left(2N_f\right)^{c\gamma} \tag{2}$$

where $\Delta\gamma$ is shear strain range, $\sigma_{n,\max}$ is normal shear stress on the plane from planes having maximum $\Delta\gamma$, k is material sensitivity factor, σ_y is yield strength, , N_f is fatigue life, τ_f', γ_f', G, b_γ, c_γ are material properties having their usual meanings.

Smith, Watson [25] developed a fatigue model for materials that predominantly fail by crack growth on the planes of maximum tensile strain and stress. The proposed relationship includes both the cyclic strain range and the maximum stress, as in Eq. (3). For multiaxial loading, the SWT parameter is based on the principal strain range $\Delta\varepsilon_1$, and maximum stress on the principal strain range plane, $\sigma_{n,\max}$. The stress term is used for describing multiaxial loading and non-proportional hardening effects.

$$\sigma_{n,\max}\frac{\Delta\varepsilon_1}{2} = \frac{\sigma_f^{'2}}{E}\left(2N_f\right)^{2b} + \sigma_f^{'}\varepsilon_f^{'}\left(2N_f\right)^{b+c} \tag{3}$$

Liu [27] presented a virtual strain energy (VSE) model which is a critical plane model, as work quantities are defined for specific planes within the material. For multiaxial loading, VSE considers two possible failure modes: a mode for tensile failure and a mode for shear failure. Failure is expected to occur on the plane having the maximum VSE quantity. ΔW_I is computed by first identifying the plane with the maximized axial work and then shear work is added on that same plane, as in Eq. (4). Similarly, ΔW_{II} is computed by first identifying the plane where shear work is maximized and then adding the axial work component as in Eq. (5).

$$\Delta W_I = \left(\Delta\sigma_n\Delta\varepsilon_n\right)_{\max} + \left(\Delta\tau\Delta\gamma\right) = 4\sigma_f^{'}\varepsilon_f^{'}\left(2N_f\right)^{b+c} + \frac{4\sigma_f^{'2}}{E}\left(2N_f\right)^{2b} \tag{4}$$

$$\Delta W_{II} = \left(\Delta\sigma_n\Delta\varepsilon_n\right) + \left(\Delta\tau\Delta\gamma\right)_{\max} = 4\tau_f^{'}\gamma_f^{'}\left(2N_f\right)^{b\gamma+c\gamma} + \frac{4\gamma_f^{'2}}{G}\left(2N_f\right)^{2b\gamma} \tag{5}$$

Chu [28] proposed a similar parameter to combine shear and normal work. To include the effects of mean stress, the author replace the stress ranges with maximum stresses, as shown in Eq. (6).

$$\Delta W = \left(\sigma_{n,\max}\frac{\Delta\varepsilon}{2} + \tau_{n,\max}\frac{\Delta\gamma}{2}\right)_{\max} = 1.02\frac{\sigma_f^{'2}}{E}\left(2N_f\right)^{2b} + 1.04\sigma_f^{'}\varepsilon_f^{'}\left(2N_f\right)^{b+c} \tag{6}$$

From the study of the above models, normal ($\Delta\varepsilon$) and shear ($\Delta\gamma$) strain ranges are identified so plasticity induced during the applied loading can be captured. The maximum and mean normal and shear stresses ($\sigma_{\max}$, $\tau_{\max}$, σ_m, τ_m) are selected so that the mean stress effects and material hardening behavior can be included during fatigue life estimation. The developed model is studied for the zero and positive mean, in-phase and out-of-phase tension torsion loading, against experimental fatigue life on a standard notched specimen.

Proposed Multiaxial Fatigue Models

Fatigue failure can be considered as the final result of a complex interaction of load and time, where loads may be monotonic, steady, variable, uniaxial or multiaxial [2]. It is characterized by crack nucleation and crack propagation, which can occur in one or more points of the structural component up to the final collapse [4]. Cracks start on the localized shear plane at or near to high stress concentrations, such as persistent slip bands, inclusions, porosity, or discontinuities. The localized shear plane usually occurs at the surface or within grain boundaries. Once nucleation occurs and cyclic loading continues, the crack tends to grow along the plane of maximum shear stress and through the grain boundary [1]. In the present model, material is assumed to be failed as nucleation starts, and a crack is formed and propagates to failure on a characteristic plane known as the critical plane identified as per determined criteria, for example, the plane having the maximum strain or stress range, maximum value of a pre-determined parameter or having maximum variance of shear stress, etc. The proposed evolution equations for the newly proposed model are formulated on a similar pattern to the endurance function model proposed by Brighenti and Carpinteri [4], with one equation defining the damage criteria parameter P, Eq. (7), expressed in terms of strain ranges ($\Delta\gamma$, $\Delta\varepsilon$), maximum shear and maximum normal stresses (τ_{max}, $\sigma_{n,max}$) determined on the critical plane.

$$P = a_1\left(\Delta\gamma.\tau_{max}\right) + a_2\left(\Delta\varepsilon.\sigma_{n,max}\right) - \frac{\sigma_L^{\ 2}}{E} - ev \tag{7}$$

where a_1, a_2 and σ_L are material constants determined by calibrating the model against a known fatigue life and load history.

Parameter *ev* in Eq. (8) defines the modification of parameter P with stress history. This can be understood with the concept of isotropic and kinematic hardening of materials. The parameter is defined with the criterion that if during a load step D>0, then *ev* is defined by Eq. (8) and if D<0 then *ev* for that step is zero (*ev*=0), which means that there is no modification in parameter P during that load step by *ev*.

$$ev = V.dP^m\left(\frac{\tau_{max} - \tau_m}{E}\right)\left(\frac{\sigma_{max} - \sigma_m}{E}\right) \tag{8}$$

where V is the material parameter and m is the power of the increment of parameter P.

The expression relating damage D_n for each load step with damage parameter P and ΔP, i.e. change in P with each load step, is shown in Eq. (9), where K and R are material parameters.

$$D_n = K.P^R.\Delta P \tag{9}$$

A location most susceptible to fatigue failure is monitored for damage accumulation, the current stress strain state at each load step is evaluated with respect to the mathematical function P (Eq.(7)) and the stress strain state leading to P < 0 is considered as this will not do any damage to the material. In other words, damage increment will occur only when parameter P for a stress strain state leads to P >1. Also, for damage increment or deterioration to occur, $\Delta P > 0$. The damage D at the point of the component

under study is evaluated as the accumulated function of damage increments D_n at each load step, so at each load step of the fatigue process, the damage increment is equal to or greater than zero, i.e. D_n, and consequently the material damage D is a non-decreasing positive function during the load history [19]. Complete damage will occur when D reaches unity (D=1). The logic flow chart of the fatigue life estimation process by the calibrated proposed model is shown in Figure 1.

Model Calibration using Genetic Algorithm

The proposed model can be used to estimate fatigue life for complex in-phase and out-of-phase loading conditions, if all parameters are calibrated according to a known loads and fatigue life. In the present section, a genetic algorithm (MOGA-II) is used to calibrate the model parameters (a_1, a_2, σ_L, V, m, K, R). GAs have advantages with respect to classical techniques, as they allow us to handle problems with multiple minima and non-convexity properties, thus avoiding numerical instability and missing the global optimum [30]. GAs operate by simulating the natural evolution process of life [31]. These algorithms are used to minimize or maximize an objective function chosen to solve a given problem. This method can be used to optimize the model parameters according to the known loading with the fatigue life for the applied loading [32, 33]. The advantage of using a GA for the proposed model is that the GA can handle any kind of objective function and can handle large population sets. It uses basic concepts like random number generation, choice, switching and combinations of such generated numbers, to get a new population which performs better than the previous generation [34]. This process is repeated iteratively until the required tolerance is achieved and thus the optimal condition can be achieved [35]. In the present study, the objective function is defined as damage prediction error err, as in Eq. (10):

$$err = D - DFL \tag{10}$$

where D is the total cumulative damage after each load step and DFL is the inverse of fatigue life at the known loading which is being used for calibration. The values of the model parameters used for characterization of the damage mechanics approach are now found by minimizing the objective function err using the GA procedure [36].

Finite Element Analysis

Stress / strain distribution is determined for each set of applied load by performing Finite Element Analysis (FEA) of the specimen geometry which is used in determining the experimental fatigue life of steel alloy C40 published in the literature [11], shown in Figure 2, with material properties given in Table 1. The finite element model is developed utilizing the ANSYS software with 10-node tetrahedral elements, to better capture the curved surfaces of the specimen geometry [37]. Dense mesh at the notch root is maintained by the sphere of influence technique. A non-linear analysis is performed as plastic properties are also included in material model of C40. Force and moment loads are applied such that they will generate normal and shear stress at the net area as mentioned in Table 1, in-phase and out-of-phase loading conditions are considered, and the corresponding force and moment and fixed support constraints are applied on the specimen model as shown in Figure 2. Details of the applied loads and respective fatigue life taken from the literature [11] are reported in Table 2. As the notch root is considered

to be the critical point for crack initiation and to predict the fatigue life as close to the experimental values as possible, the state of stress and strain at the notch root is used in estimation of fatigue life.
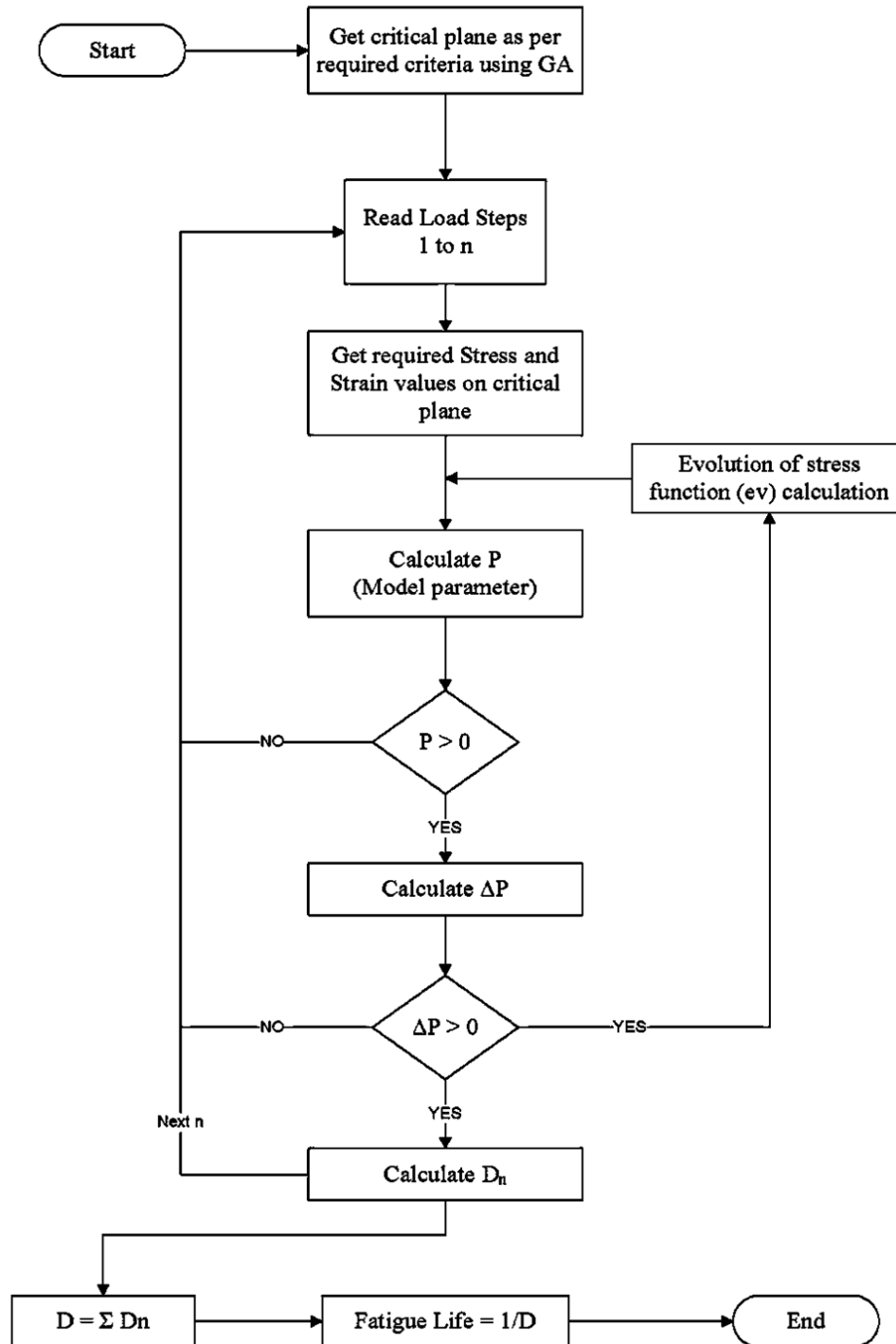

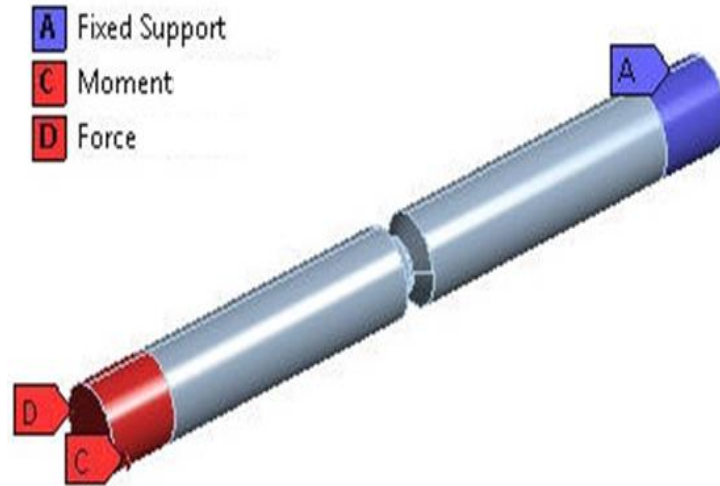Figure 1. Fatigue life estimation process flow chart for proposed model.

Figure 2. C40 test specimen's FEA model and applied loads on specimen.

Table 1. Mechanical properties of C40 steel.

Material name	Young's Modulus [38]	Yield stress (MPa)	Ultimate tensile strength (MPa)	Cyclic strain hardening exponent, n'	Cyclic strength coefficient, K' (MPa)
C40a	206	537	715	0.131b	915b

Source: (a) [11];   (b) SAE [39].

RESULTS AND DISCUSSION

A steel alloy C40 specimen (Figure 2) is tested against in-phase and out-of-phase loading conditions published in the literature (Table 2). The proposed model is implemented in ANSYS, by developing an algorithm using Ansys Parametric Design Language (APDL). The algorithm identifies the critical plane by identifying the plane with the maximum value of the critical plane variable crtpl, as in Eq. (11). Normal and shear stress and strain ranges ($\sigma_{n,max}$, τ_{max}, $\Delta\varepsilon$, $\Delta\gamma$) with respect to the critical plane are calculated. Now, for calibration loads CP_1 and CP_2 (Table 2), for each set the calculated stresses and strains are used to determine the model parameters using GA procedures (MOGA-II). The fatigue life value is calculated for the remaining loads other than that used for calibration, using the proposed damage evolution equations, Eqs. (7–9). To implement this, code is developed in APDL which uses the model parameters calculated earlier and estimates fatigue life for the current applied loading.

$$crtpl = \left(\Delta\gamma.\tau_{max}\right) + \left(\Delta\varepsilon.\sigma_{n,max}\right) \tag{11}$$

The proposed model is implemented by performing FEA to calculate the stress state at the notch root surface of the specimen, as the notch root is the most susceptible region for failure. Then stress components on the critical plane are calculated and used with Eqs. (7)– [29] and the GA tool to estimate the coefficients of the proposed model to estimate the fatigue life. Two sets of fatigue life are predicted: one with one calibration point CP_1 and the second with two calibration points CP_1 and CP_2 (Table 3), where the

coefficients for each load step are interpolated between the coefficients of the calibration points.

Table 2. Experimental loads and fatigue life of C-40 specimen having notch radius 0.5 mm ($\sigma_a / \tau_a = 1$) [11].

Normal and shear stress at net area (σa, τa) (MPa)	Load ratio (R)	Phase difference (°)	Cycles to failure (N_f) $\times 10^6$
101$_{(CP2)}$	-1	0	2.0
129.75	-1	0	0.18
159.92	-1	0	0.072
179	-1	0	0.21
200$_{(CP1)}$	-1	0	0.027
221	-1	0	0.012
99.6$_{(CP2)}$	-1	90	2.0
119.5	-1	90	0.94
140	-1	90	0.285
160.25	-1	90	0.044
180	-1	90	0.016
199.7$_{(CP1)}$	-1	90	0.011
67.9$_{(CP2)}$	0	0	2.0
79.72	0	0	0.79
99.56	0	0	0.35
119	0	0	0.157
138.58	0	0	0.0477
158.1$_{(CP1)}$	0	0	0.026
66.8$_{(CP2)}$	0	90	2.0
89.55	0	90	0.34
119.3	0	90	0.094
138.75	0	90	0.034
158.1$_{(Cp1)}$	0	90	0.022

Note: CP$_1$ and CP$_2$ are first and second calibration points for model coefficients determination

Results from experimental fatigue life values from the literature (Table 2) [40] and predictions made by the proposed model with one and two calibration points are reported in Figure 3. The results show that the proposed model predicted results for R = -1 (zero mean) load with phase = 0 with good agreement for the single calibration point (CP$_1$) as well as the two calibration point case (CP$_1$ & CP$_2$). The CP$_1$ only case is on the slightly conservative side, but the two-point case follows the experimental fatigue life trend more closely, although slightly on the non-conservative side but in an acceptable range, and can be considered as predicting the fatigue behavior with acceptable accuracy. Similarly, for R = -1 (zero mean) load with phase = 90, with the CP$_1$ only case the results are increasingly on the conservative side as we move away from the calibration point load. But for the two-point CP$_1$ & CP$_2$ case, the predicted fatigue life agreed well with the experimental fatigue life. From this we see the effectiveness of using more than one calibration point and the results are found to be on the conservative side but in range to

be declared as in good agreement with the experimental life. Brighenti, Carpinteri [41] also reported that for different materials the endurance function model, which has a similar structure to the evolution equations in the newly proposed model in this study, behaves close to experimental results with loading conditions having R = -1.
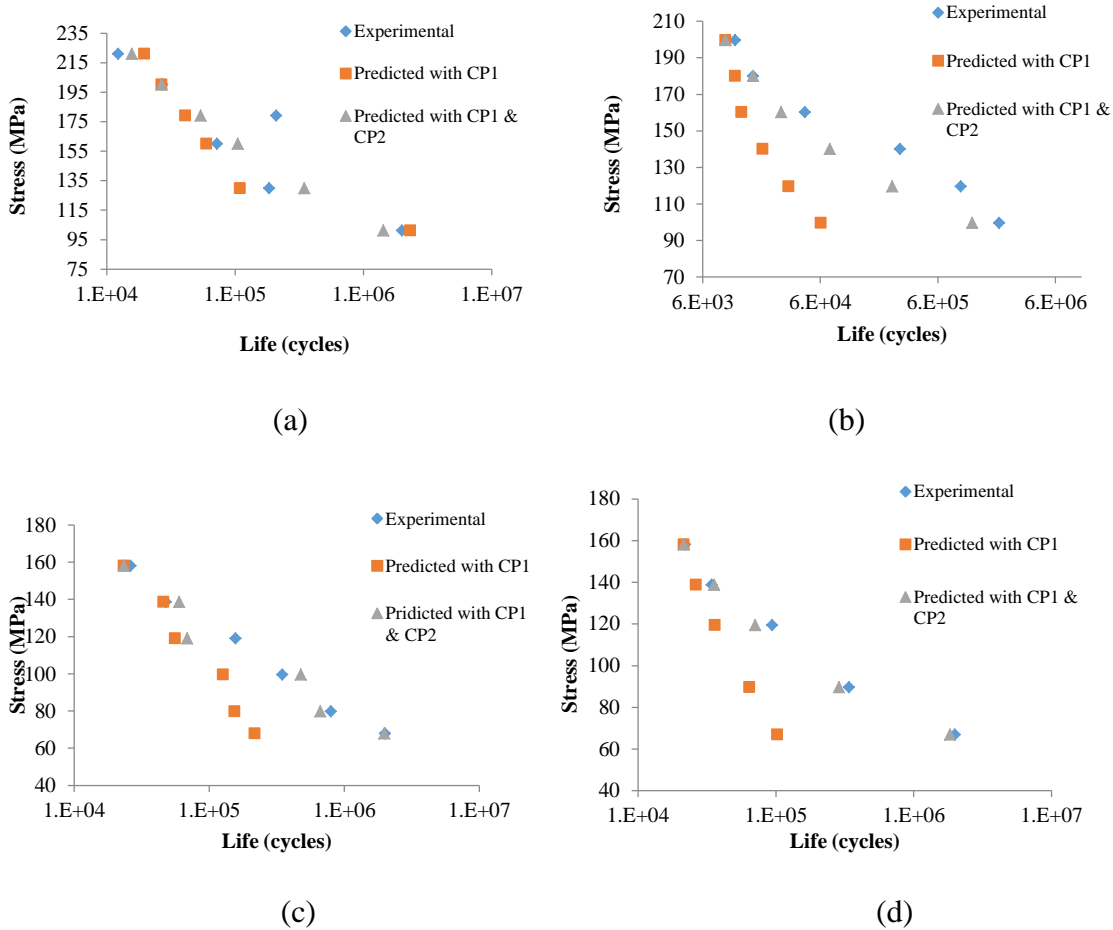

(a)

(b)

(c)

(d)

Figure 3. Predicted fatigue life (a) for R = -1 and phase = 0°, (b) for R = -1 and phase = 90°, (c) R = 0 and phase = 0°, (d) R = 0 and phase = 90°.

Now for the cases of R = 0 (positive mean) for both the phase = 0 and phase = 90 cases, the CP_1 only case results in a diverging trend of fatigue life as we move away from the calibration point. The predicted life is on the conservative side but noticeably different from the experimental fatigue lives as we move from the low-cycle to high-cycle region. Similarly, when we analyze the case with CP_1 and CP_2 we can see that the predicted fatigue life is in very good agreement with the experimental fatigue life, on the slightly conservative side, but it follows the trend of experimental life with appreciable accuracy, thus again showing the effectiveness of more than one calibration point for the proposed model. From the above results, the effectiveness of the proposed model is clearly shown in proportional as well as non-proportional loading cases. Also, the importance of more than one load point to calibrate the proposed model is highlighted. The application of GA to estimate the coefficients of the proposed mode equations includes an inherent flexibility to accommodate any kind of loading conditions. In this study we took the values of normal and shear stress to be the same, i.e. $\sigma a / \tau a = 1$, which made the interpolation task for model coefficients relatively simple. In future studies the proposed model will be tested for cases with different load values for normal and shear

stresses and also a methodology to interpolate the model coefficients in that case. Also, the proposed model will be tested against more complex loading conditions.

Table 3. Predicted fatigue life of C-40 steel.

Normal and shear stress at net area (σ_a, τ_a) (MPa)	Experimental cycles to failure (N_f) x10^6	Predicted cycles to failure (N_p)x10^6	
		One calibration point (CP$_1$)	Two calibration points (CP$_1$ & CP$_2$)
R = -1 and phase = 0			
101	2.0	2.32	1.43
129.75	0.18	0.109	0.347
159.92	0.072	0.06	0.105
179	0.21	0.041	0.054
200	0.027	0.0268	0.0268
221	0.012	0.02	0.0157
R = -1 and phase = 90			
99.6	2.0	0.06	1.182
119.5	0.94	0.032	0.24
140	0.285	0.02	0.072
160.25	0.044	0.0128	0.028
180	0.016	0.0113	0.016
199.7	0.011	0.009	0.0094
R = 0 and phase = 0			
67.9	2.0	0.217	1.98
79.72	0.79	0.15	0.664
99.56	0.35	0.12	0.478
119	0.157	0.056	0.069
138.58	0.0477	0.046	0.06
158.1	0.026	0.0234	0.023
R = 0 and phase = 90			
66.8	2.0	0.102	1.84
89.55	0.34	0.064	0.287
119.3	0.094	0.036	0.07
138.75	0.034	0.026	0.036
158.1	0.022	0.021	0.0215

CONCLUSIONS

A new fatigue life estimation model has been proposed using the continuum mechanics concepts. Experimental fatigue lives for steel alloy C40 for in-phase and out-of-phase loading conditions are used to calibrate and analyze the accuracy of the proposed model. The new model predicted fatigue life with good agreement for in-phase and out-of-phase loads for zero mean and positive mean of loading stresses when two calibration points were used. But a diverging trend from experimental life was shown when moving away from the calibration point for the single calibration point case. Overall, the model has shown good accuracy and is simple to apply. A more detailed study is needed to examine

the performance of the new model against more complex multiaxial and variable amplitude loading conditions.

ACKNOWLEDGMENTS

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