Adaptive Nonlinear Control of AC-DC-PWM Converter
Applied for Induction Heating

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Abstract – In this paper an adaptive nonlinear control strategy applied to a PWM converter is developed and simulated. First a nonlinear system modelling is derived with state variables of the input current and the output voltage. The system is linearized and decoupled, and then a state feedback law is obtained. For robust control to parameter perturbation, an adaptive nonlinear control strategy is introduced in order to assure tracking and regulation of the output DC voltage and increase the power factor of the installation composed from DC-DC-AC feeding an induction heating. The simulation was performed to verify the feasibility of the adopted control algorithm.

I. INTRODUCTION

In recent year, three-phase voltage source PWM converters are increasingly used for applications such as UPS systems, electric traction and induction heating. The attractive features of them are constant DC bus voltage, low harmonic distortion of the utility currents, bidirectional power flow and controllable power factor [1- 5].

Yakoubi et al. analyzed a design and performance of voltage and current PI controllers, which are composed of an inner current control loop and outer voltage control loop in a cascade structure [6]. In [7,8], the PWM converter has been modelled in a nonlinear system using power balance concept between the input terminal and the output one. In [9], the nonlinear systems were analyzed and the controllers were designed using small signal analysis which is valid only around operating points, on which linear control is based.

In this paper, an adaptive nonlinear control technique for a PWM converter associated with a power circuit of the quasi resonant DC link (QRDCL) converter [10-13] is investigated. We first show that it is feasible to apply nonlinear multiple input-multiple output (MIMO) feedback linearization technique to such a system that is operated in high frequency regimes. Also, the effect of parameter perturbation on the control performance is investigated. In order to eliminate the steady state error, an adaptive control is also introduced to feedback control law.

II. MODELLING OF THE PROPOSED CONVERTER

A power circuit of PWM three phase voltage source AC-DC converter is introduced in figure 1. This circuit could be modelised and an equivalent circuit is derived figure 2 [9].

It is assumed that an equivalent resistive load \( R_{\text{load}} \) is connected to the output terminal, this load based on simplified model of the DC-DC-AC converter is given in figure 3. The nominal model of the proposed converter in a synchronous reference frame is given by:

\[
\begin{aligned}
\frac{d i_{\text{sd}}}{dt} &= -\frac{R}{L} i_{\text{sd}} + \omega i_{\text{sq}} + \frac{1}{L}(e_{\text{sd}} - v_{\text{sd}}) \\
\frac{d i_{\text{sq}}}{dt} &= -\frac{R}{L} i_{\text{sq}} - \omega i_{\text{sd}} + \frac{1}{L}(e_{\text{sq}} - v_{\text{sq}})
\end{aligned}
\]

(1)

where \( e_{\text{sd}} \) and \( e_{\text{sq}} \) are the d-q axis source voltage, \( i_{\text{sd}} \) and \( i_{\text{sq}} \) the d-q axis source current, \( v_{\text{sd}} \) and \( v_{\text{sq}} \) are the d-q axis converter input voltage. The parameters \( R \) and \( L \) represent the line resistance and the input inductance, respectively. The index \( n \) denotes nominal values.

III. DESIGN OF THE ADAPTIVE CONTROLLER

Our objective is to design an adaptive nonlinear controller for use with a PWM converter having parameter but unknown \( R_{\text{load}} \). We start by designing a nonlinear input-output linearizing controller for the model of the PWM converter. We can rewrite the system of equations (1) in a form that suggests an adaptive scheme for the estimation of \( R_{\text{load}} \).

Fig. 1. Power circuit of PWM converter.

Fig. 2. Per-phase equivalent circuit.
Fig. 3. A power circuit of PWM converter associated with a power circuit of the QRDCL converter feeding induction heating.

\[ x = f_0(x) + g(x)u + \delta f'(x), \]  

where

\[ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad x_1 = i_{sd}, \quad x_2 = i_{sq}, \quad x_3 = V_C, \]  

and

\[ u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad u_1 = e_{sd} - v_{sd}, \quad u_2 = e_{sq} - v_{sq}, \]  

\[ f_0 = \begin{bmatrix} -\frac{R}{L}i_{sd} + \omega i_{sq} \\ -\frac{R}{L}i_{sq} - \omega i_{sd} \\ 2\frac{e_{sd}i_{sd} + e_{sq}i_{sq}}{3VC} - \frac{V_C}{R_{load}C} \end{bmatrix}, \]  

\[ f_d = \begin{bmatrix} 0 & 0 & -\frac{V_C}{C} \end{bmatrix}^T, \]  

\[ \delta \] represents the error between the inverse of the uncertain parameter (\( R_{load} \)) and its nominal inverse value, i.e.:

\[ \delta = \frac{1}{R_{load}} - \frac{1}{R_{load,n}} = \theta - \theta_n, \]  

A. Non-adaptive version of the controller

We start by designing the non-adaptive version of the controller, where \( \delta \) is assumed to be zero. The control goal is to regulate the output voltage \( V_C \). The outputs are therefore chosen as \( [y_1, y_2]^T \) where \( y_1 = i_{sd} \) and \( y_2 = V_C \). We apply the feedback linearization technique which uses a nonlinear change of coordinates and feedback to transform the nonlinear system (2) into a decoupled linear one [9]. Let the nonlinear change of coordinates be (see appendix):

\[ [z] = [z_1, z_2, z_3]^T = [y_1, y_2, y_2]^T, \]  

then:

\[ [z_1] = A(x) + E(x)[u_1, u_2]^T, \]  

The linearizing control law is then:

\[ [u] = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = E^{-1}(x)(-A(x) + v), \]  

where \( v = [v_1, v_2]^T \) is the new system input vector to be determined. Classical pole placement scheme can be used to insure output voltage tracking:

\[ \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_{1ref} - k_{11}e_1 \\ y_{2ref} - k_{21}e_1 - k_{22}e_2 \end{bmatrix}, \]  

where

\[ e_1 = y_1 - y_{1ref} \] and \( e_2 = y_2 - y_{2ref} \).

We can determine by simulation the poles that give good performance. By using MATLAB Control System Toolbox, one can obtain the matrix gain k.

B. Adaptive version of the controller

Since \( \delta \) is unknown, we replace it by its estimate:

\[ \dot{\delta} = \frac{1}{R_{load}} - \frac{1}{R_{load,n}} = \dot{\theta} - \dot{\theta}_n, \]  

and by replacing \( z \) in (7) and \( u \) in (9) by \( \dot{z} \) and \( \dot{u} \) which depend on the parameter estimate (11). The output vector is: \( [y_1, y_2]^T \). The nonlinear change of coordinates (7) gives:
\[
\begin{bmatrix}
\hat{z}_1 \\
\hat{z}_2 \\
\hat{z}_3
\end{bmatrix} = A_\delta + E(x) \begin{bmatrix}
\hat{u}_1 \\
\hat{u}_2
\end{bmatrix},
\]
(12)

\[
\begin{bmatrix}
\hat{z}_1 \\
\hat{z}_2 \\
\hat{z}_3
\end{bmatrix} = A_{\delta_1}(x) + A_{\delta_2}(x) \hat{\theta} + E(x) \begin{bmatrix}
\hat{u}_1 \\
\hat{u}_2
\end{bmatrix},
\]
(13)

where \( A_\delta \) is the estimation of \( A_0 \).

\[
A_\delta(x) = A_{\delta_1}(x) + A_{\delta_2}(x) \hat{\theta},
\]
(14)

Linearizing control law becomes:

\[
\begin{bmatrix}
\hat{u}_1 \\
\hat{u}_2
\end{bmatrix} = E^{-1}(x)(-A_{\delta_1}(x) - A_{\delta_2}(x) \hat{\theta} + \hat{\nu}),
\]
(15)

The new system inputs \( \hat{u}_1 \) and \( \hat{u}_2 \) are designed exactly as (10). In closed loop, the system (12) becomes:

\[
z = y + \delta W,
\]
(16)

where

\[
W = -A_{\delta_2}(x),
\]
(17)

We define the vector of the states error by

\[
e = \begin{bmatrix}
e_1 \\
e_2 \\
e_3
\end{bmatrix},
\]
(18)

which satisfies:

\[
\dot{e} = -k e + \delta^T W,
\]
(19)

The parameter adaptation laws and the control of the system can be obtained as follows:

\[
\dot{\delta} = -\gamma W^T e,
\]
(20)

\[
u = E^{-1}(x)(-A_\delta(x) + \hat{\nu}),
\]
(21)

where \( \gamma \) is the adaptation gain vector.

Finally, we note that we estimate the inverse of the uncertain parameter not the parameter itself. We can obtain the error between the estimated and the actual value. A block diagram of the adaptive scheme used is given in Fig. 4.

IV. SIMULATION RESULTS

The performances of the closed loops system have been evaluated at the nominal power operation using Matlab simulation tool. Only linear load with uncertainty was tested. The simulation of the steady state operation was performed at nominal power \( P_n = 10 \text{ kW} \) and the parameters are given in Table 1.

Figure 5 and figure 6 show that the voltage \( V_C \) is controlled, in the case of a step change of the load, with the desired values. Figure 7 shows the estimation of the parameter \( R_{\text{Load}} \) which follow the simulated values. Figure 8 shows that the source power factor is controlled as unity as usual.

It is observed that the supply current is close to sinusoidal and it remains in phase with the supply voltage, therefore, unity power factor is maintained at the output of supply system.

![Block diagram of the adaptive scheme used](image)

![Fig. 5. Transient responses of the capacitor voltage \( V_C \) for step change of load](image)

![Fig. 6. Zoom of the capacitor voltage \( V_C \)](image)
V. CONCLUSION

In this paper a nonlinear MIMO state space mathematical model of the PWM converter by feedback linearization is developed. Moreover, an adaptive control is designed in order to diminish the influence of the unknown load uncertainties and disturbances and also to reduce the number of sensors in the system. The proposed control scheme gives satisfactory simulation results with nominal load.

VI. REFERENCES


VII. APPENDIX

\[
f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{V_c}{C} \end{bmatrix} \theta ,
\]

\[
A_c(x) = \frac{2}{3CV_c} (e_d f_1 + e_q f_2) - \frac{2}{3CV_c^2} (e_d j_d + e_q j_q) + \frac{1}{R_{load} C} f_1,
\]

\[
E(x) = \begin{bmatrix} \frac{1}{L} e_d \\ \frac{1}{2L} e_d \end{bmatrix}, \quad A_{cs}(x) = \begin{bmatrix} 0 \\ -\frac{f_1}{C} \end{bmatrix}.
\]
\[ A_{s1}(x) = \begin{bmatrix} \frac{f_1}{2} \left( e_{sd} f_1 + e_{qf} f_2 \right) - \frac{2}{3C V_c} \left( e_{sd} i_{sd} + e_{qf} i_{qf} \right) f_3 \end{bmatrix} \]

### Table I

PARAMETER OF THE PWM CONVERTER

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply’s Voltage &amp; frequency</td>
<td>220V, 50Hz</td>
</tr>
<tr>
<td>Line’s inductance &amp; resistance</td>
<td>0.1mH, 2mΩ</td>
</tr>
<tr>
<td>DC link resistance</td>
<td>20Ω</td>
</tr>
<tr>
<td>Output capacitor</td>
<td>370 μF</td>
</tr>
<tr>
<td>PWM carrier frequency</td>
<td>1KHz</td>
</tr>
</tbody>
</table>